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METHOD OF SPLITTING OF PHYSICAL FACTORS FOR NONSOLENOIDAL MOTION OF GAS-LIQUID MEDIA

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A generalization of the method of splitting of physical factors for the case of nonsolenoidal motion of gas-liquid media is proposed. Use of the method is demonstrated by the example of calculation of the dynamics of a melt in a ladle during its blow-through via a submergible lance.

Introduction. The nonsolenoidal character of the motion of gas-liquid media, i.e., the nonzero divergence of the barycentric velocity $\nabla \cdot \mathbf{V} = \Phi \neq 0$, is its most distinctive feature [1]. Within the "vacuum" approximation (when $\rho'/\rho_0 \to 0$)

$$\Phi = \Psi - \alpha d'(\ln \rho')/dt - \nabla \cdot (\alpha \mathbf{W}) , \tag{1}$$

and therefore the nonsolenoidal character of the motion of gas-liquid media can be related, first, to the presence of volume gas-phase sources Ψ (the first term), second, to its compressibility (the second term, proportional to $d'(\ln \rho')/dt$), and, third, to interphase motion (the last term, proportional to $\nabla \cdot (\alpha \mathbf{W})$).

Investigation of the mass-transfer processes in a ladle during injection treatment of a metal melt is an example of a practical problem that requires taking account of the nonsolenoidal character of the motion [2]. In this case, all three above-mentioned factors play important roles: a rather intense gas source exists at the feeder, the gas bubbles expand considerably during their floating up, and intense relative motion of this liquid and gas phases is observed. Numerous examples of problems of the type can be adduced, although the methods of their solution are usually based on a Boussinesq approximation in which the nonsolenoidal character is neglected [3-5], which is not always correct. In [5], effects of the compressibility of the gas-liquid medium are accounted for only in the equation for gas-phase transfer.

In what follows, we generalize the method of splitting of physical factors [6] to the nonsolenoidal case. With this generalization, we investigate the motion of the gas–melt medium during the blow-through of metal in a 350-ton ladle with argon via a submergible lance.

Method. The method was published for the first time in [7], where it was presented for the general case of a multiphase medium with account for heat-transfer processes. In the present work, the particular case of gas–liquid media with small values of the gas-content coefficient α is considered within the "vacuum" approximation. In addition, it is assumed for simplicity that, under the conditions being considered, the density of the gas phase depends only on the pressure p (the temperature T and the other parameters ζ entering into the equation of state for the gas $\rho' = \rho'(p, T, \zeta)$ are fixed). In this case, the dynamics of the gas–liquid medium is described by the system of equations

$$\partial \mathbf{V}/\partial t = R(\mathbf{V}, \alpha) - \nabla \tilde{p} , \tag{2}$$

$$\nabla \cdot \mathbf{V} = \Phi(\mathbf{V}, \alpha) , \tag{3}$$

$$\partial \alpha/\partial t = -\nabla \cdot (\alpha \mathbf{V}) + \Phi(\mathbf{V}, \alpha) , \tag{4}$$

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where

$$R(\mathbf{V},\alpha) = -(\mathbf{V}\cdot\nabla)\mathbf{V} + \nu_e \Delta \mathbf{V} + \zeta_e \nabla(\nabla\cdot\mathbf{V}) + (1-\alpha)\mathbf{g}\,. \tag{5}$$

Due to the above assumptions, $d'(\ln\rho')/dt = \xi d'p/dt$, where $\xi = \partial(\ln\rho')/\partial p$. For a polytropic process, $\xi = 1/\gamma p$. By neglecting the dynamic component of the pressure as compared to the static one (which is justified for flow regimes realized during the blow-through [5]), we obtain $d'p/dt = \rho_0 \mathbf{g}\cdot\mathbf{V}'$, from which it follows that

$$\Phi(\mathbf{V},\alpha) = \Psi - \alpha\,\xi\,\rho_0\,\mathbf{g}\cdot(\mathbf{V}+\mathbf{W}) - \nabla\cdot(\alpha\mathbf{W})\,. \tag{6}$$

The diffusion velocity $\mathbf{W}$ of the gas medium entering into this expression has collective $\mathbf{W}^c$ and turbulent $\mathbf{W}^t$ components. The first one $\mathbf{W}^c = -W^c\mathbf{g}/g$ can be found by one of the empirical formulas of [8] or set up based directly on experimental data. The second component $\mathbf{W}^t$ is determined by the effective coefficient of turbulent diffusion of the gaseous phase: $\alpha\mathbf{W}^t = -D_e\nabla\alpha$.

According to the method of splitting of physical factors [6], the term containing the pressure in Eq. (2) is split off at each time step τ:

$$\tilde{\mathbf{V}} = \mathbf{V}^n + \tau R(\mathbf{V},\alpha)\,, \tag{7}$$

$$\mathbf{V}^{n+1} = \tilde{\mathbf{V}} - \tau\nabla\tilde{p}\,. \tag{8}$$

In order to obtain an equation for the pressure, we make use of relationship (3) and require that it be satisfied exactly on the $n+1$-th time layer

$$\nabla\cdot\mathbf{V}^{n+1} = \Phi(\mathbf{V}^{n+1},\alpha^{n+1})\,. \tag{9}$$

By taking the divergence of both sides of Eq. (8) and taking Eq. (9) into account, the following relationship is obtained on the $n+1$-th time layer:

$$\Delta\tilde{p}^{n+1} = [\nabla\cdot\tilde{\mathbf{V}} - \Phi(\mathbf{V}^{n+1},\alpha^{n+1})]/\tau\,. \tag{10}$$

If explicit and implicit calculation schemes are chosen for Eqs. (7) and (4), respectively, we obtain a system of equations for evaluation of the characteristics of the motion of the medium:

$$\text{I}\quad \tilde{\mathbf{V}} = \mathbf{V}^n + \tau R(\mathbf{V}^n,\alpha^n)\,, \tag{11}$$

$$\text{II}\quad \alpha^{n+1} = \alpha^n + \tau\,[-\nabla\cdot(\alpha^{n+1}\mathbf{V}^{n+1}) + \Phi(\mathbf{V}^{n+1},\alpha^{n+1})]\,, \tag{12}$$

$$\Delta\tilde{p}^{n+1} = [\nabla\cdot\tilde{\mathbf{V}} - \Phi(\mathbf{V}^{n+1},\alpha^{n+1})]/\tau\,, \tag{13}$$

$$\mathbf{V}^{n+1} = \tilde{\mathbf{V}} - \tau\,\nabla\tilde{p}^{n+1}\,. \tag{14}$$

In the first stage (explicit calculations), the intermediate velocity $\tilde{\mathbf{V}}$, for which Eq. (3) is generally not satisfied, is found without taking the pressure $\tilde{p}$ into account. In the second stage (implicit calculations), we find the gas content α^{n+1} and the pressure $\tilde{p}^{n+1}$, which serves for correcting the intermediate velocity $\tilde{\mathbf{V}}$ to a value $\mathbf{V}^{n+1}$ satisfying relationship (3), since the equation for the pressure has been found from the condition of satisfying this relationship.

In the second stage, one has to solve the system of interrelated equations (12)-(14). To do this, we use an iterative method. However, to avoid solving the Poisson equation (13) at each iteration step k, we replace it by the corresponding evolution equation

$$\tilde{\tilde{p}} = \tilde{p} + \omega \left\{ \Delta \tilde{p} - [\nabla \cdot \tilde{\mathbf{V}} - \Phi(\mathbf{V}, \alpha)]/\tau \right\} \tag{15}$$

with a certain evolution parameter ω providing convergence of the iterative process (15) [9], and we solve all the equations of the second stage within a single iterative cycle, which makes it possible to reduce the computation time substantially. As a result, we obtain the following scheme of splitting of physical parameters for nonsolenoidal motion of a gas–liquid medium:

$$\text{I} \quad \tilde{\mathbf{V}} = \mathbf{V}^n + \tau R(\mathbf{V}^n, \alpha^n), \tag{16}$$

$$\alpha^{n+1,0} = \alpha^n, \quad \tilde{p}^{n+1,0} = \tilde{p}^n, \quad \mathbf{V}^{n+1,0} = \mathbf{V}^n, \tag{17}$$

$$\text{II} \quad \alpha^{n+1,k+1} = \alpha^n + \tau [-\nabla \cdot (\alpha^{n+1,k} \mathbf{V}^{n+1,k}) + \Phi(\mathbf{V}^{n+1,k}, \alpha^{n+1,k})], \tag{18}$$

$$\tilde{p}^{n+1,k+1} = \tilde{p}^{n+1,k} + \omega [\Delta \tilde{p}^{n+1,k} - (\nabla \cdot \tilde{\mathbf{V}} - \Phi(\mathbf{V}^{n+1,k}, \alpha^{n+1,k}))/\tau], \tag{19}$$

$$\mathbf{V}^{n+1,k+1} = \tilde{\mathbf{V}} - \tau \nabla \tilde{p}^{n+1,k+1}. \tag{20}$$

Since the second stage of the scheme is performed within an iterative cycle, it would be advisable to free it from all calculations that can be performed in the first stage. Thus, the term $\nabla(D_e \nabla \alpha)$, connected with the turbulent diffusion of the gas phase (and entering into $\Phi(\mathbf{V}, \alpha)$), can be split off Eq. (17) and calculated by the explicit scheme in the first stage, and the calculation of the contribution of convective terms and sources (introducing the strongest instability in the calculations) is carried out in the second stage by the implicit scheme.

It should be noted that, if necessary, calculation of the temperature can be included in scheme (16)-(20) by analogy with the calculation of the gas-phase concentration, which permits taking into account the temperature dependence in the equation of state for the gas.

Adequacy of the Method and Example of the Calculation. In the case of conditions where factors leading to nonsolenoidal motion of gas–liquid media become important, direct experimental data on the hydrodynamics and distribution of the gas phase are virtually absent. These conditions are realized, e.g., during blow-through of a melt in a ladle with an inert gas. In this case, indirect experimental data are available, and we intend to use them to verify the adequacy of our method.

Calculations were carried out for a 350-ton ladle with the following geometric parameters: mean radius $R = 1.8$ m and metal height $H = 4.9$ m. The lance, situated on the ladle axis, is submerged in the metal to a depth of 4.5 m. The effective gas flow was chosen within the limits of 40–100 m^3/h. A value of 0.5 m/sec was assumed for W^c [10].

For the velocities on all solid surfaces and the symmetry axis, boundary conditions of impenetrability and free slipping were chosen, and the boundary condition of free flow was chosen on the free surface. Boundary conditions for the pressure were obtained by projecting Eq. (20) onto the normal to the surface. Boundary conditions for the gas-content coefficient on all the surfaces were chosen in the form $\mathbf{n} \cdot \nabla \alpha = 0$, which corresponds to impenetrability of the gas through the free surface of the melt. It was assumed that a gas-phase source Ψ that is determined by the gas flow through the lance operates in the zone of formation of the bubbling regime.

Due to the cylindrical symmetry, the problem can be formulated in two dimensions in cylindrical coordinates. To model the turbulence, a three-parameter algebraic model was used, as in [5]. In addition, it was assumed that $\zeta_e = 0$.

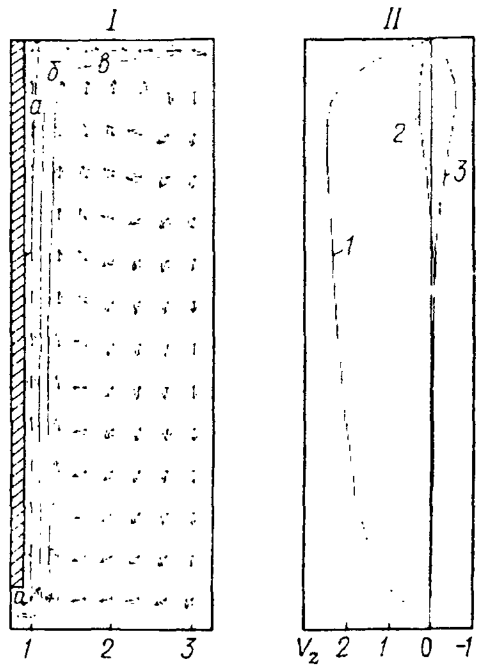

Fig. 1. Hydrodynamics of a melt in a ladle in blow-through via a submergible lance. V_z, m/sec.

Results of calculations are illustrated by Fig. 1, corresponding to the variant with a gas flow of 60 m^3/h. Figure 1 *I* presents calculated velocity direction fields of the melt and isolines of the volume concentration of the gas phase corresponding to the following values of α: a) 0.5, b) 0.2, c) 0.05. Figure 1 *II* presents dependences of vertical velocity components on the height at different distances from the axis; the numbers at the curves in Fig. 1 *II* correspond to the numbers in Fig. 1 *I*.

The overall picture of the motion and distribution of the gas phase corresponds to that observed in experiments and agrees with data obtained by other methods [3, 5, 10], which bears witness to the qualitative adequacy of our method in the given case.

The maximum velocity of the medium is observed in the vicinity of the lance (curve 1 in Fig. 1 *II*) near the melt surface and reaches 2.5 m/sec in the given regime.

The gas phase moves mainly in the region adjacent to the lance (Fig. 1 *I*). At a distance of 0.3 m from the lance, the volume concentration of the gas already does not exceed 1% virtually along the entire lance length except for the near-surface region, where gas inclusions are deflected from the lance by turning melt flows.

In all blow-through regimes, $\alpha > 0.5$ in the immediate vicinity of the lance both at its lower portion (where the gas is supplied) and at the near-surface portion, which is related to expansion of the gas bubbles upon their floating up.

One can trace the dependence of the gas-content coefficient on the height in the immediate vicinity of the lance for various blow-through regimes with the help of the plots presented in Fig. 2. The z axis corresponds to the distance from the ladle bottom; the quantity h_{lan} marks the location of the lower cross section of the lance. The letters at the curves in Fig. 2 correspond to different gas flows: a) 40, b) 60, c) 80, d) 100 m^3/h. High values of the gas content correspond to the zone of formation of the bubbling regime, since here the gas-liquid medium still has a relatively low velocity and the gas has no time to spread (see Fig. 1 *II*). Then the medium gains velocity and the gas-content coefficient falls. However, beginning with values of z corresponding to approximately one-quarter of the ladle height, an increase in the gas content is observed despite the fact that the velocity of the medium still continues to increase. This rise in the plots of α is connected with both the effect of expansion of gas inclusions upon reaching higher horizons and interphase motion and is not observed when these factors are neglected, although it does take place in practice.

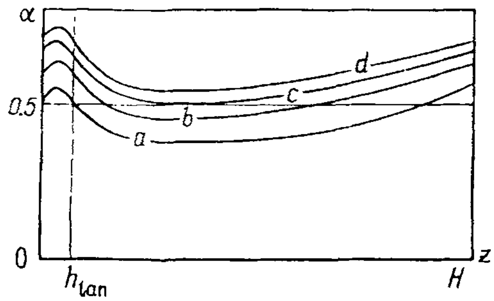

Fig. 2. Dependence of the gas-content coefficient on the height in the immediate vicinity of the lance.

Regions with a high gas content are undesirable in blow-through. Indeed, at high values of α (say $\alpha > 0.5$) there is a high probability of formation of continuous gas channels through which the gas being supplied exits directly to the atmosphere, which leads to a sharp decrease in the effect of melt mixing. In addition, when powdered admixtures are fed into the ladle along with the gas, they can be transported pneumatically to the atmosphere and be lost [11]. On the other hand, the gas supply should be fast enough to ensure mixing of the melt in the ladle and reliable pneumatic transport of the powder through the lance.

The value $\alpha^* = 0.5$ is taken as the maximum permissible value of the gas-content coefficient. Then a continuous zone with a gas content exceeding α^* is formed along the lance at a gas flow rate of 100 m^3/h (the entire curve d lies above the value α^*). At a gas flow rate of 80 m^3/h, the zone with $\alpha > \alpha^*$ near the lance has a discontinuity, but on an insignificant length (curve c), and therefore the value of 80 m^3/h can be considered, from our point of view, as the limiting admissible value. At a gas flow rate of 40 m^3/h, the intensity of mixing is extremely low. Thus, we conclude that gas flow rates equal to approximately 60 m^3/h are optimum from the viewpoint of mixing of the melt in the ladle and prevention of formation of gas channels.

The results presented are substantiated by investigations under industrial conditions [11], which indicates indirectly the quantitative adequacy of our model and the method used for its construction.

To reveal the effect of various factors of the nonsolenoidal character of the motion on results of calculations of the blow-through of a melt in a ladle, we performed test computations: 1) in the solenoidal approximation (with account for all factors only in the gas-phase transfer equation (2), as in [5], 2) with the compressibility of the gas phase and the interphase motion being neglected (only the contribution of the source Ψ was taken into account), 3) with the interphase motion being neglected. These computations have shown that, in the case under consideration, the source term Ψ is the most important factor of the nonsolenoidal character of the motion. Qualitatively, it determines the fan-like velocity distribution in the region of the gas supply, which, in turn, affects substantially both the further distribution of the gas phase in the ladle, which is introduced into the melt precisely in this place, and the velocity values. Whereas in variant 1 the maximum value of the velocity of the medium equals ~0.7 of the corresponding value in the original variant, a value of ~0.85 is obtained for variant 2, and already a value of ~0.95 is obtained in variant 3. Similar relationships occur for the gas-content coefficients.

CONCLUSIONS

1. The numerical method proposed describes adequately the nonsolenoidal motion of gas–liquid media within the "vacuum" approximation.

2. In investigations of the motion of a melt in a ladle, neglect of the nonsolenoidal character of the motion leads to a qualitatively incorrect description of the velocity direction field in the region of the gas supply and to quantitative errors in the velocities and gas-content coefficients that can be as large as 30%.

3. In the blow-through regimes considered, the factors of the nonsolenoidal motion can be placed in order of decreasing importance as follows: gas sources, compressibility of the gas phase, and interphase motion.

4. The regime with a gas flow rate of ~0.60 m^3/h is most rational from the viewpoint of melt mixing and prevention of formation of gas channels in a 350-ton ladle.

NOTATION

V, barycentric velocity of the medium; **W**, diffusion velocity of the gas medium; $d'/dt = \partial/\partial t + \mathbf{V}'\cdot\nabla$, substantial derivative along the velocity of the gas phase $\mathbf{V}'$; $\widetilde{\mathbf{V}}$, auxiliary velocity; g, free-fall acceleration; ρ_0, ρ_0', densities of the liquid and gas phases; p, pressure; $\widetilde{p} = p/\rho_0$; α, gas-content coefficient; ν_e, ζ_e, effective coefficients of viscosity of the liquid and gas; D_e, effective diffusion coefficient; Ψ, volume density of the gas-phase sources; R and H, radius and height of the ladle; t, time; τ, time step; γ, polytrope exponent; **n**, unit vector of the normal to the surface.

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